SIMPLE HARMONIC MOVEMENT

Introduction to the Simple Harmonic Movement

This page is for the student to observe the representation of the Simple Harmonic Movement (S.H.M. from now on), identifying the principle magnitudes which intervene, and visualizing the values of these in different cases, as well as the variations which they experience at different instants and positions.

OBJECTIVES

- To identify the S.H.M. as a periodic, oscillatory and vibratory movement.
- To visualize a body which describes an S.H.M.
- To define and identify the principle physical magnitudes which intervene in an S.H.M.
- To visualize and interact with the graphs which represent these magnitudes.
- To visualize the relation which exists between the S.H.M. and the Uniform Circular Movement.

1.1 Definitions

A **periodic** movement is one in which its characteristic magnitudes are repeated at regular intervals of time. A periodic movement is **oscillatory** if it covers the trajectory in both directions. An oscillatory movement is **vibratory** if its trajectory is rectilinear and it is centred on the origin.

A **HARMONIC** movement is a vibratory movement in which position, velocity and acceleration can be described with sinusoidal (sine or cosine) functions. Of all harmonic movements, the simplest one is the Simple Harmonic Movement which is the one we will be referring to from now on.

A **SIMPLE HARMONIC MOVEMENT** is one in which the position of the body is given by a function of the following type.

$$y = A \cdot sin(\omega \cdot t + \varphi)$$

By clicking on **next** you will see the meaning of each one of the magnitudes which appear in this equation

Fundamental magnitudes

Elongation (y): is the distance from the moving body to the origin (O) of the movement at any instant.

Amplitude (A): is the maximum elongation.

Period (T): is the time it takes to make a complete vibration.

Frequency (f): is the number of complete vibrations made in a unit of time . It is

the reciprocal of period: $$f = \frac{1}{T}$$

Angular frequency or pulsatance (ω):

$$\omega = 2\pi f = \frac{2\pi}{T}$$

Phase difference or phase angle(φ): is the value which determines the position of the body at the start of the movement (when time equals zero). We will see its meaning later on.

1.2 Representing the S.H.M.

By clicking on next you can see the representation of a body describing an S.H.M. and observe the meaning of the previously mentioned magnitudes.

To make things simpler at the beginning, it is assumed that in this particular case, there is no phase difference, that is φ=0. In this case the equation of the movement takes the form:

y= A · sin (ω·t)

1.3 Representation of an S.H.M.
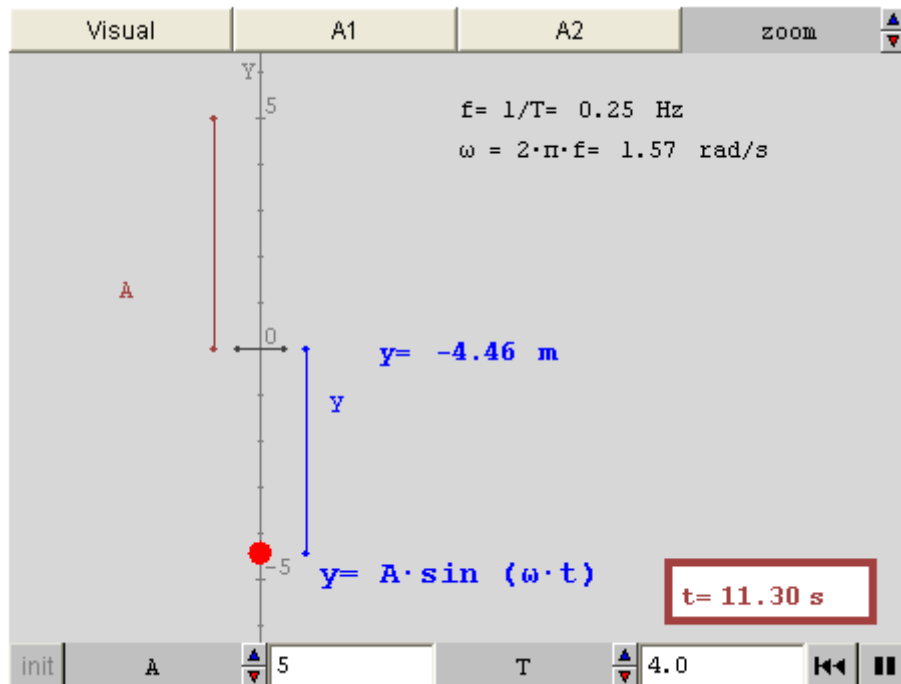
Visual: This visual represents a body that describes an S.H.M. in the simplest case (when there is no phase difference).

A1: Change the value of the amplitude with the A control and watch the result. Use the zoom control to see the complete movement if necessary.

A2: Change the value of the period with the T control and watch the change in the movement. Observe the variation in the values of the frequency and the angular frequency shown on the screen as the period changes.
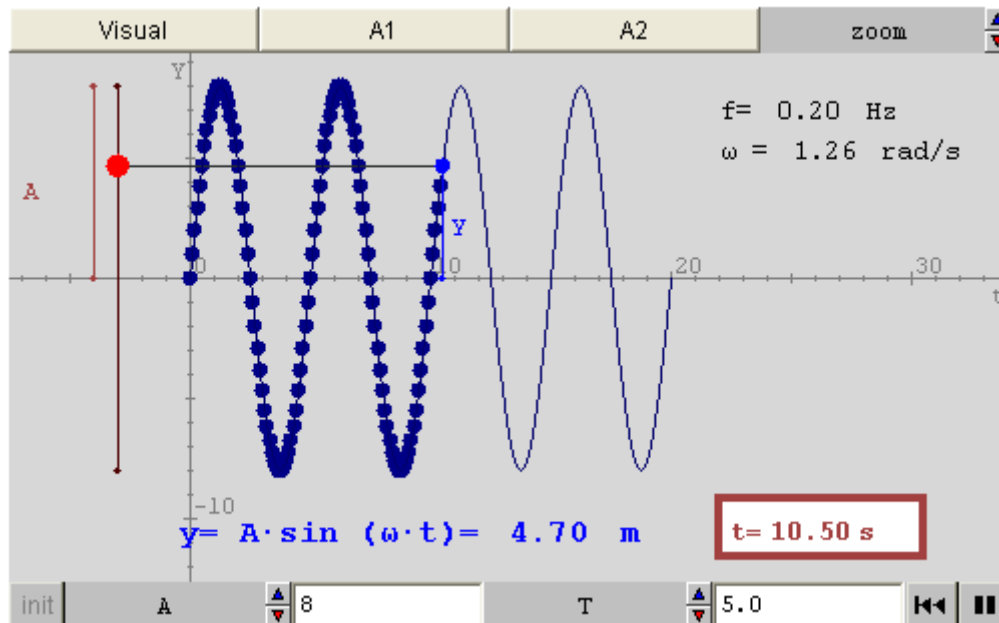
2.1 Position in the S.H.M.

As mentioned, the position of a body which describes an S.H.M. is given by a sinusoidal type equation:

$$y = A \cdot sin(\omega \cdot t)$$

The simplest case occurs when there is no phase difference ($\varphi = 0$). In this case the equation is reduced to:

$$y = A \cdot sin(\omega \cdot t + \varphi)$$

2.1 Position of a body in an S.H.M.

Visual: In this visual you can see how the position moves along a sinusoidal (sine) trajectory as time goes by.

A1: Change the value of the amplitude and observe the changes in the graph.

A2: Now change the period of the movement and analyse the changes in the graph, the angular frequency and the frequency.
 Would you be able to identify the value of the period from the graph?
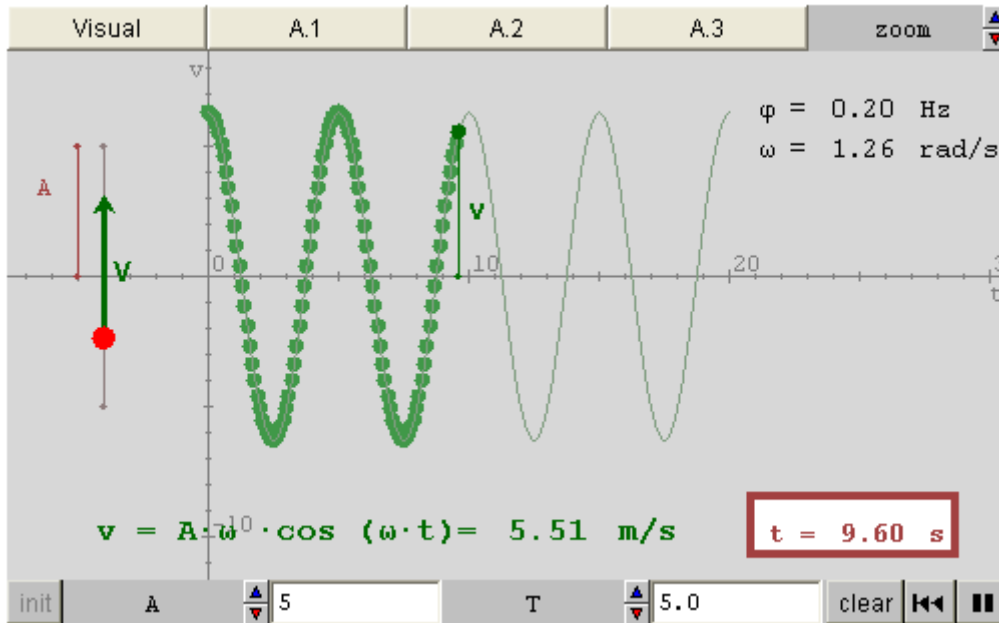
2.2 Velocity in the S.H.M.

The velocity of a moving body which describes an S.H.M. is obtained from the derivative of the position with respect to time:

$$v = \frac{dy}{dt} = A \cdot \omega \cdot \cos(\omega \cdot t + \varphi)$$

If we again keep to the simplest case, in which the **phase difference φ= 0 ,** the equation is simplified:

$$v = A \cdot \omega \cdot \cos(\omega \cdot t)$$

Visual: This visual represents the velocity vector of the body moving with an S.H.M.
 Velocity versus time is represented in the graph (a cosine function).

A1: By changing the values of the amplitude and the period, you can see their influence on the velocity vector and the graph that represents it.

A2: The velocity vector is always tangent to the trajectory and it points in the same direction as the movement. When will the velocity have a positive sign in the graph? When will it have a negative sign?

A3: Investigate the movement of the body and the graph to determine the position of the body when the speed is maximum and minimum.
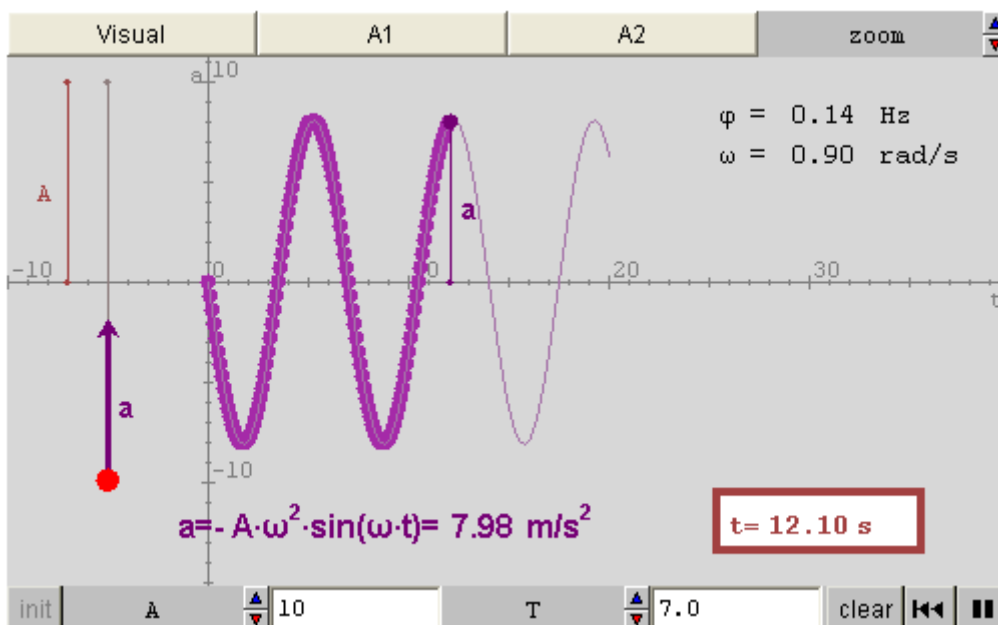
2.3 Acceleration in the S.H.M.

As the S.H.M. is a rectilinear movement, its normal acceleration is zero.

Therefore, the total acceleration coincides with the tangential acceleration and thus can be obtained from the derivative of the speed.

$$a = \frac{dv}{dt} = -A\cdot\omega^2\cdot\sin(\omega t + \varphi)$$

In the simplest case, the phase difference is nil ($\varphi = 0$) and the equation takes the form:

$$a = -A\cdot\omega^2\cdot\sin(\omega t)$$

Visual: This visual represents the acceleration vector.
 The graph represents the value of acceleration (vertical axis) versus time (horizontal axis).

A1: The acceleration vector is always tangent to the trajectory in this movement and its orientation depends on the elongation. When is the sign of the acceleration positive? When is it negative?

A2: By looking at the graph and the vector, determine the positions in which the acceleration is maximum and minimum.

2.4 The relation between the S.H.M. and the Uniform Circular Movement

The S.H.M. of a **point** can be considered as the "*projection*" (the projected shadow) of an **auxiliary point** describing a uniform circular movement (U.C.M.) <u>with a radius equal to the amplitude A and an angular velocity equal to the angular frequency ω</u>. The real body can be considered to be moving along the vertical diameter of the circumference covered by the auxiliary body.
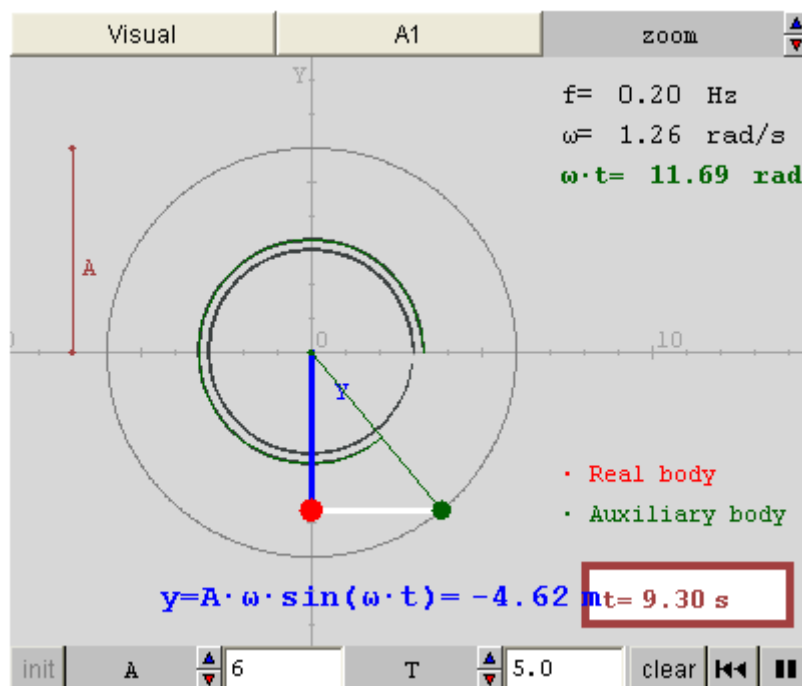
Visual: Click on play to start the movement with the default amplitude and period. Observe the fact that the position of the real body coincides with the shadow of the auxiliary body which would be caused by spotlights on either side of the visual on the vertical diameter of the circumference.

A1: Change the values of the amplitude and the period and observe the changes in the movement. Use the zoom controls if necessary. Observe the fact that the product ω•t coincides with the angle covered by the auxiliary body at every instant.

2.5 The meaning of the phase difference

Until now we have always considered that the real moving body begins its movement (at t = 0 s) at the coordinate origin (y = 0 m) and moves towards the positive ordinates (upwards). However, the initial situation may be different, and to reflect this fact we introduce the concept of the **phase difference "φ".**

Remember that the complete equation of an S.H.M. is:

$$y = A \cdot sin(\omega \cdot t + \varphi)$$

The function of the phase difference"φ" is to indicate what the position of the moving body is at the start (when t= 0) and in which direction it is moving.

To determine its value we must "imagine" what position the auxiliary body must be in at the beginning so that, once the movement has started, its shadow continues to be on the real body, as we saw when we studied the relation between the S.H.M. and the U.C.M.
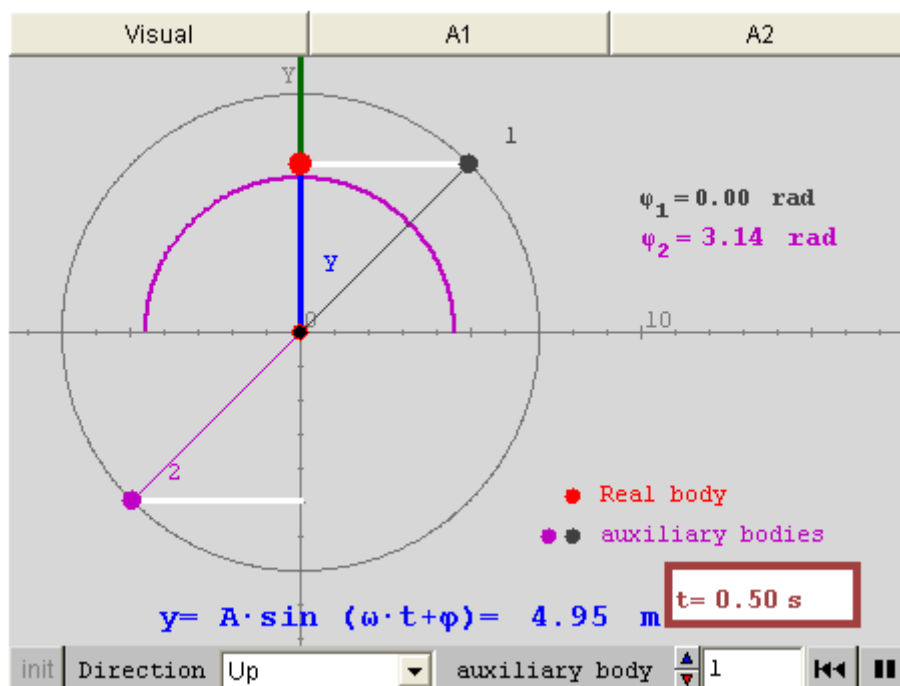
Visual: Drag the real body to the desired initial position on its trajectory.
 Observe the auxiliary bodies

A1: Choose the initial direction of the movement from the menu. Choose the right auxiliary body for the movement from the auxiliary body menu and click on play to see if you made the right choice. Repeat the experiment a couple of times choosing the initial position of the real body and the direction in which it moves.

A2: Note that when the body is at the edge of its trajectory the position of the two auxiliary bodies coincides. Therefore, either of them can be chosen and the example no longer illustrates the point.

2.6 Conclusions about the kinematics of an S.H.M.

- The Simple Harmonic Movement is a periodic movement in which the position varies according to a sinusoidal (sine or cosine) equation.

- The velocity of the body continually changes, being maximum at the centre of the trajectory and nil at the limits, where the body changes the direction of the movement.

- The S.H.M. is a **non-uniformly accelerated movement**. Its **acceleration is proportional to displacement** and in the opposite direction. It shows its maximum value at the limits and its minimum value in the centre.

- We can imagine an S.H.M. as the projection of a Uniform Circular Movement. The phase indicates the position of the body at the start when time equals zero.

3.1 The origin of the S.H.M.: Elastic force

As we have seen before, when we studied acceleration in the S.H.M.	$a = -A \cdot \omega^2 \cdot sen(\omega \cdot t + \varphi)$

If this is written in respect of position:	$a = -\omega^2 \cdot y$

$$F = -m \cdot \omega^2 \cdot y = -k \cdot y$$

By applying Newton's second law we obtain the value for **elastic force:**

where $k = m \cdot \omega^2$ is called *the elastic constant of the movement*, and is measured in N/m.

Visual	A1	zoom

k= 4.93 N/m
y= 7.00 m
a= -17.27 m/s^2

F= -k·y= -34.54 N

y= A·sin (ω·t+φ)   t= 5.00 s

init   A   7   T   4.0   mass   2
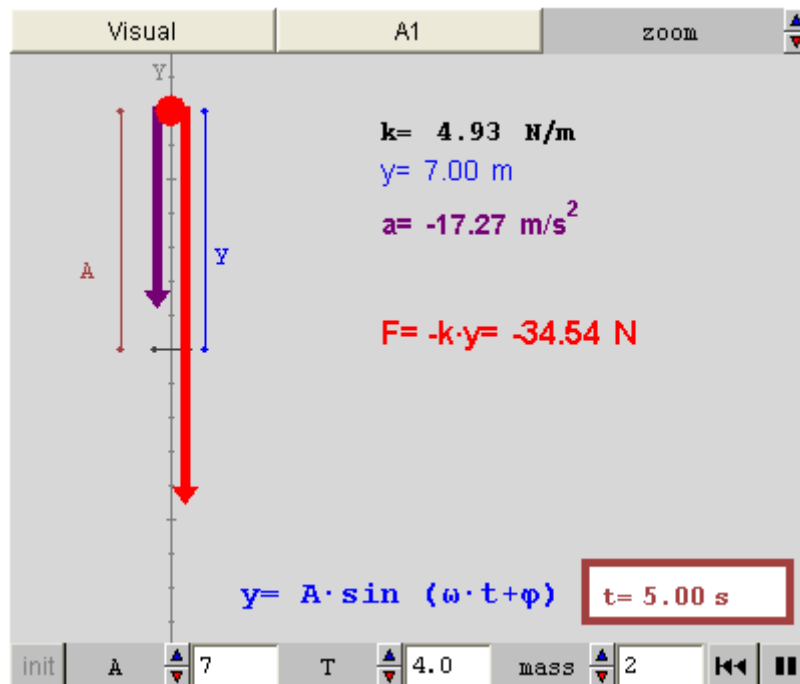
Visual: By clicking on play you can see the acceleration vector (purple) and the elastic force vector (red).
 Select the values of the amplitude, the period and the mass with the controls. Use the zoom control if necessary.

A1: Stop the animation at different positions and establish the relation between the direction of the acceleration and force and the sign of the elongation.
 Identify the position of the body when the values of the elastic force are maximum and minimum.
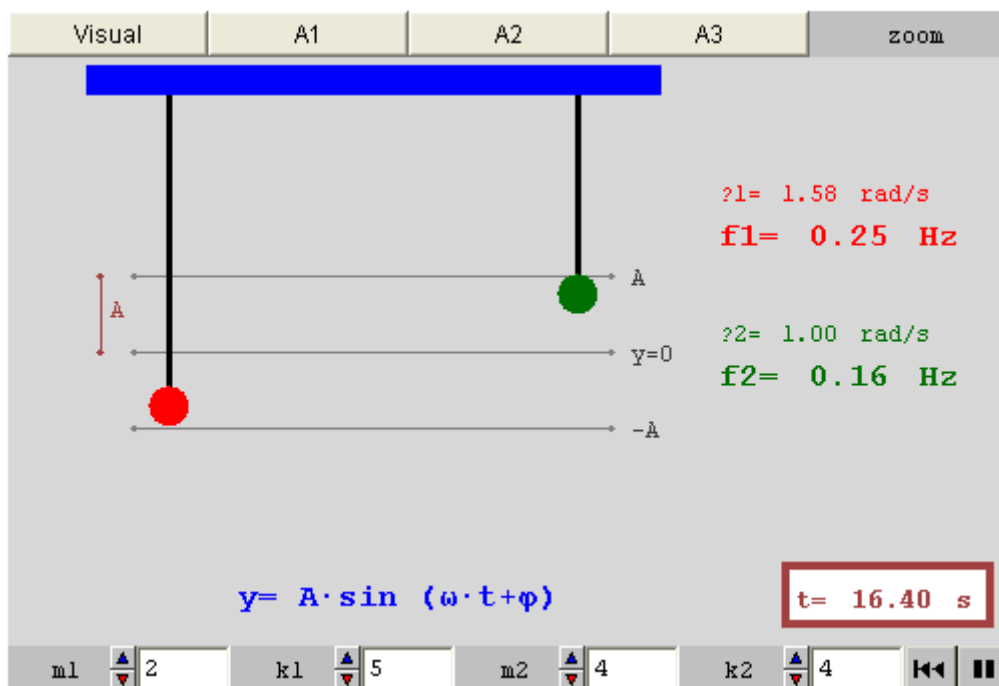
Frequency of the vibration.

Using the definition of the elastic constant, we obtain the angular frequency:

$$\omega = \sqrt{\frac{k}{m}}$$

And remembering the relation between angular frequency and frequency, we get:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

You can see that frequency <u>depends exclusively on the elastic constant of the movement and the mass</u> of the body describing it. On the following page we try to show the influence of both magnitudes on the frequency of the movement:

Visual	A1	A2	A3	zoom

?1= 1.58 rad/s
f1= 0.25 Hz

?2= 1.00 rad/s
f2= 0.16 Hz

y= A·sin (ω·t+φ)

t= 16.40 s

m1	2	k1	5	m2	4	k2	4

Visual: In this visual, the black vertical segments represent springs on which the red body (1) and the green body (2) are hanging. Before clicking on play, drag the red body downwards. Note that the amplitude of the movement depends on this action. Then click on play.

A1: Restart the visual. Use the same elastic constant for both springs, but choose different masses for the bodies.
Note down your conclusions.

A2: Now use equal masses and different elastic constants for the springs.
Observe the frequencies and note down your conclusions.

A3: Keep the masses equal to each other and try to find values of k1 and k2 that make one frequency double the other frequency. ¿What is the relation between these constants?
Now use the same constant for both springs and try to find values of the masses that make one frequency double the other frequency.

3.3 Conclusions about the Dynamics of the S.H.M.

- The **elastic force** responsible for an S.H.M. is always **contrary and proportional to the displacement**.

- The **frequency** of the vibration of a body describing an S.H.M. **depends solely on its mass and the elastic constant**, while being independent of the amplitude of the vibration.

4.1 Mechanical energy is conserved in the S.H.M.

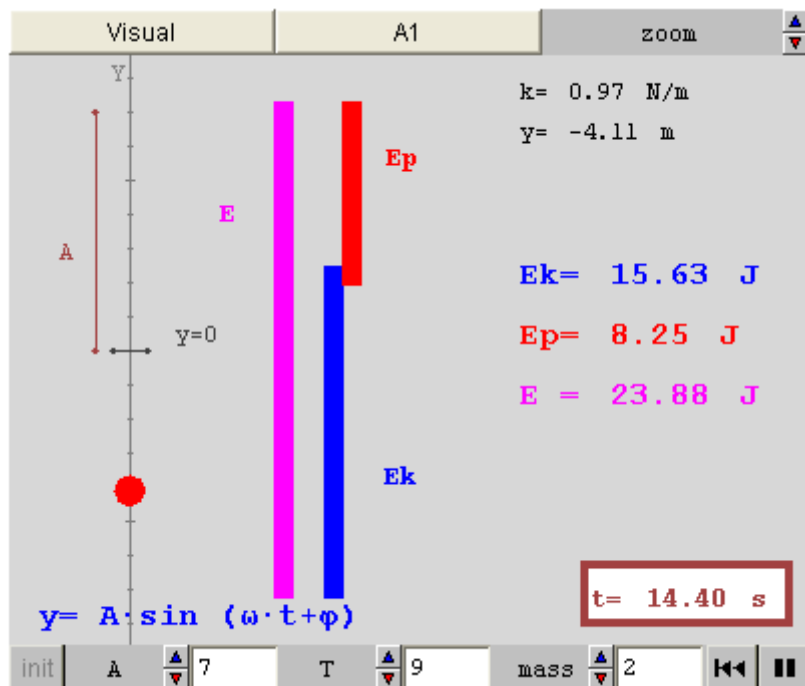Any moving body possesses **kinetic energy**. If velocity is written with regard to position, we get:

$$E_k = \frac{1}{2} k (A^2 - y^2)$$

Moreover, given that the **elastic force acting on the body is a conservative force**, the body carries with it a certain amount of **potential elastic energy**, according to the expression:

$$E = E_k + E_p = \frac{1}{2} k \cdot A^2$$

 The values of kinetic and potential energy depend on the position occupied by the body. However, **the total energy possessed by the body remains constant during the whole trajectory**.

$$E_p = \frac{1}{2} k \cdot y^2$$

Visual	A1	zoom

k= 0.97 N/m
y= -4.11 m

Ep

E

A

y=0

Ek= 15.63 J

Ep= 8.25 J

E = 23.88 J

Ek

t= 14.40 s

y= A·sin (ω·t+φ)

init   A   7   T   9   mass   2

Visual: When you click on play, you will see the kinetic energy represented by a blue stripe, the potential energy by a red stripe and the total energy by a pink stripe.
 Observe the fact that, although the potential and kinetic energy constantly change their value, their sum remains constant.

A1: The kinetic and potential energies change as the position of the body changes.
 Note down the positions at which the potential and kinetic energies are maximum and minimum.
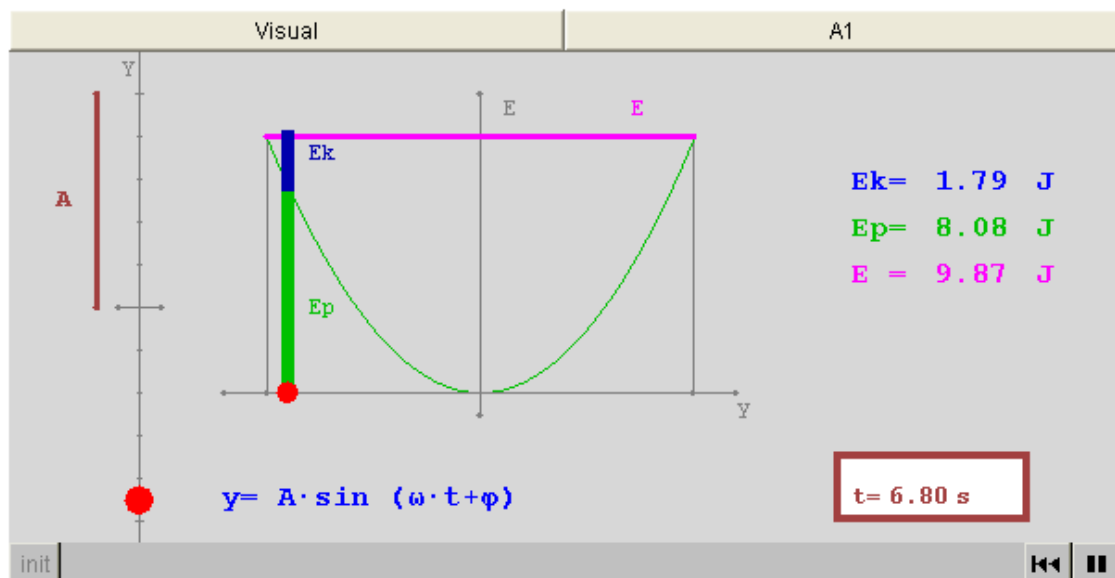
4.2 Graphs of the energies involved

There is another way to represent the variations produced in the kinetic and potential energies of an S.H.M.

Remember that the expression for potential energy is:

$$E_p = \frac{1}{2}k \cdot y^2$$

and therefore its graph corresponds to a parabola.

Mechanical or total energy remains constant and therefore its graph corresponds to a straight line.

Visual: In this visual the energies are represented on the vertical axis and the elongation is represented on the horizontal axis. The green parabola represents the potential energy and the pink line represents the total energy of the body.

The difference between the potential energy (green segment) and the total energy (pink) is equal to the kinetic energy.

A1: Check the fact that the total energy remains constant even though the potential and kinetic energies constantly change. Find the positions of the body that make both types of energy maximum and minimum.

4.3 Conclusions about Energy in the S.H.M.

- **The elastic force which produces an S.H.M. is conservative.** The potential elastic energy which it carries with it is nil in the centre of the trajectory and maximum at its limits.

- Kinetic energy in the S.H.M. continually varies, being maximum in the centre of the trajectory and nil at the limits.

- Given the conservative nature of the elastic force, **the total mechanical energy of the body remains constant throughout the whole trajectory.**

5.1 EVALUATION
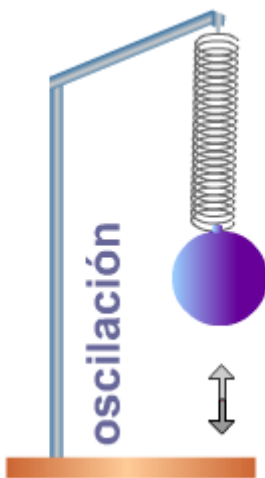
Click on next and you will find a series of questions which will permit you to evaluate your knowledge of some of the concepts dealt with in this teaching unit.

1 In order to determine the phase of the movement, we must know

☐ the amplitude and the direction of the movement.

☐ the position of the body at the start of the movement (when time equals zero).

☐ the position and the direction of the body at the start of the movement (when time equals zero).

☐ the frequency and the initial position.

2 The speed at which the body moves

☐ increases when the body moves in one direction and decreases when it moves in the opposite direction.

☐ is maximum at the centre of the trajectory and zero at the limits.

☐ is proportional to the elongation.

☐ remains constant during the movement.

3 A Simple Harmonic Movement can be classified as

☐ a uniform rectilinear movement.

☐ a non-uniformly accelerated rectilinear movement.

☐ a uniformly accelerated rectilinear movement.

4 What does the phase indicate?

☐ The situation of the body at the start of the movement (when time equals zero).

☐ The frequency of the movement.

☐ The time it takes the body to start moving.

5 In an S.H.M.

☐ the kinetic and potential energies remain constant through the whole process.

☐ the mechanical energy remains constant.

☐ there is only kinetic energy, and it remains constant.

☐ the potential energy remains constant, while the kinetic energy changes constantly.

6 If we change the period of the movement we will also be changing

☐ the amplitude and the angular frequency.

☐ the frequency and the phase.

☐ the frequency and the angular frequency.

☐ the phase and the amplitude.

7 In an S.H.M. the mechanical energy is proportional to

☐ the phase.

☐ the amplitude of the movement.

☐ the square of the amplitude.

☐ the period.

8 The frequency of the S.H.M. is given by

☐ the mass of the body and the elastic constant.

☐ the phase and the elongation.

☐ the amplitude of the movement.

9 The S.H.M. is produced by the elastic force. This force

☐ is proportional to the amplitude.

☐ has a constant modulus.

☐ is inversely proportional to the phase.

☐ is proportional to the elongation, but it points in the opposite direction.

10 What is the period of an S.H.M.?

☐ The time it takes the body to go from one edge of the trajectory to the other.

☐ The time it takes to complete an oscillation.

☐ The time it takes the body to go from the center to the edge of the trajectory.

☐ The number of oscillations per unit of time.

11 The acceleration of the S.H.M.

☐ is zero at the centre of the trajectory and maximum at the limits.

☐ is maximum at the centre of the trajectory and zero at the limits.

☐ is constant everywhere in the trajectory.

☐ has the same sign as the elongation.